Novelty Assessment Report

Paper: DR-Submodular Maximization with Stochastic Biased Gradients: Classical and Quantum Gradient Algorithms
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Abstract

In this work, we investigate DR-submodular maximization using stochastic biased gradients, which is a more realistic but challenging setting than stochastic unbiased gradients. We first generalize the Lyapunov framework to incorporate biased stochastic gradients, characterizing the adverse impacts of bias and noise. Leveraging this framework, we consider not only conventional constraints but also a novel constraint class: convex sets with a largest element, which naturally arises in applications such as resource allocations. For this constraint, we propose an $1/e$ approximation algorithm for non-monotone DR-submodular maximization, surpassing the hardness result $1/4$ for general convex constraints. As a direct application of stochastic biased gradients, we consider zero-order DR-submodular maximization and introduce both classical and quantum gradient estimation algorithms. In each constraint we consider, while retaining the same approximation ratio, the iteration complexity of our classical zero-order algorithms is $O(\epsilon^{-3})$, matching that of stochastic unbiased gradients; our quantum zero-order algorithms reach $O(\epsilon^{-1})$ iteration complexity, on par with classical first-order algorithms, demonstrating quantum acceleration and validated in numerical experiments.

Core Task Landscape

This paper addresses: **DR-Submodular Maximization with Stochastic Biased Gradients**

A total of **9 papers** were analyzed and organized into a taxonomy with **8 categories**.

Taxonomy Overview

The research landscape has been organized into the following main categories:

- **Gradient Estimation and Approximation Methods**
- **Online and Sequential Optimization Frameworks**
- **Generalized Function Classes and Theoretical Extensions**
- **Application-Driven Optimization**

Complete Taxonomy Tree

- DR-Submodular Maximization with Stochastic Biased Gradients Survey Taxonomy
- Gradient Estimation and Approximation Methods
  - Zeroth-Order and Quantum Gradient Estimation ★ (2 papers)
  - [0] DR-Submodular Maximization with Stochastic Biased Gradients: Classical and Quantum Gradient Algorithms (Anon et al., 2026) View paper
  - [5] Zeroth-order stochastic approximation algorithms for DR-submodular optimization (Y Lian, 2024) View paper
  - Block-Coordinate and Projection-Free Methods (2 papers)
  - [4] Projection-Free Online Optimization with Stochastic Gradient: From Convexity to Submodularity (Chen Lin, 2018) View paper
  - [6] Stochastic Block-Coordinate Gradient Projection Algorithms for Submodular Maximization (Zhigang Li, 2018) View paper
- Online and Sequential Optimization Frameworks
  - Online Learning with Constraints (1 papers)
  - [3] A single recipe for online submodular maximization with adversarial or stochastic constraints (Omid Sadeghi, 2020) View paper
  - Regularized Online Optimization (1 papers)
  - [7] Regularized online DR-submodular optimization (P Zuo, 2023) View paper
- Generalized Function Classes and Theoretical Extensions
  - Weakly DR-Submodular Functions (1 papers)
  - [9] A Unified Approach for Maximizing Continuous Î³-weakly DR-submodular Functions (M Pedramfar, n.d.) View paper
  - Linearizable Optimization Frameworks (1 papers)
  - [1] From linear to linearizable optimization: A novel framework with applications to stationary and non-stationary dr-submodular optimization (Vaneet Aggarwal, 2024) View paper
- Application-Driven Optimization
  - Mean Field Inference and Probabilistic Models (1 papers)
  - [2] Optimal continuous DR-submodular maximization and applications to provable mean field inference (Yatao Bian, 2019) View paper
  - Information Content and Big Data (1 papers)
  - [8] Information Content of Big Data (Karbasi, 2021) View paper

Narrative

Core task: DR-submodular maximization with stochastic biased gradients. The field centers on optimizing diminishing returns submodular functions—a broad class capturing many machine learning objectives—under various computational and informational constraints. The taxonomy reveals four main branches: Gradient Estimation and Approximation Methods, which develop techniques for computing or approximating gradients when exact evaluations are costly or unavailable; Online and Sequential Optimization

Frameworks, which address settings where objectives or constraints arrive over time; Generalized Function Classes and Theoretical Extensions, which broaden the scope beyond classical DR-submodularity to weaker or related notions; and Application-Driven Optimization, which tailors algorithms to specific domains such as information gathering or resource allocation. Early foundational work like Continuous DR-Submodular[2] established convergence guarantees for continuous domains, while methods such as Projection-Free Stochastic[4] and Block-Coordinate Gradient[6] introduced scalable first-order approaches that avoid expensive projection steps or exploit problem structure.

Recent efforts have intensified around handling imperfect gradient information and adapting to dynamic environments. A small handful of works explore zeroth-order or quantum-inspired gradient estimation when first-order oracles are unavailable, with Zeroth-Order Stochastic[5] providing convergence rates under function-value queries alone. Meanwhile, online frameworks like Online Submodular Recipe[3] and Regularized Online DR[7] tackle sequential decision-making with regret guarantees, and extensions such as Gamma-Weakly DR-Submodular[9] and Linear to Linearizable[1] relax strict submodularity assumptions to accommodate broader function classes. The original paper, Stochastic Biased Gradients[0], sits squarely within the gradient estimation branch alongside Zeroth-Order Stochastic[5], addressing the practical challenge of biased stochastic gradients—a setting where noise is not zero-mean. This contrasts with earlier unbiased stochastic methods like Projection-Free Stochastic[4] and complements zeroth-order techniques by assuming gradient access but relaxing the unbiasedness requirement, thereby bridging first-order and derivative-free paradigms.

Related Works in Same Category

The following **1 sibling papers** share the same taxonomy leaf node with the original paper:

1. Zeroth-order stochastic approximation algorithms for DR-submodular optimization

Authors: Y Lian, X Wang, D Xu, Z Zhao | **Year/Venue**: 2024 | **URL**: View paper

Abstract

â¦ -wise gradient estimator and the randomized gradient â¦ We also extend NZOSA to handle a class of robust DR-submodular â¦ the variance caused by the biased integral auxiliary function. â¦

Relationship Analysis

Both papers belong to the Zeroth-Order Gradient Estimation category, focusing on DR-submodular maximization using function value queries without direct gradient access. They overlap in addressing stochastic gradient estimation for DR-submodular optimization under convex constraints, with both proposing algorithms that achieve approximation ratios near (1-1/e). The key difference is that the original paper introduces quantum gradient estimation methods alongside classical approaches and extends the Lyapunov framework to handle stochastic biased gradients, while the candidate paper focuses on classical zeroth-order methods using coordinate-wise and randomized gradient estimators with variance reduction techniques for finite-sum problems.

Contributions Analysis

Overall novelty summary. The paper extends DR-submodular maximization to handle stochastic biased gradients, generalizing the Lyapunov analysis framework and proposing algorithms for both classical and quantum zero-order settings. It resides in the 'Zeroth-Order and Quantum Gradient Estimation' leaf alongside one sibling paper (Zeroth-Order Stochastic), making this a relatively sparse research direction within the broader taxonomy of nine papers across six leaf nodes. The focus on biased gradients distinguishes it from prior unbiased stochastic methods and complements existing zeroth-order techniques by relaxing the zero-mean noise assumption while retaining gradient-based convergence analysis.

The taxonomy reveals that gradient estimation methods form one of four main branches, with the original paper's leaf sitting under 'Gradient Estimation and Approximation Methods' alongside 'Block-Coordinate and Projection-Free Methods'. Neighboring branches address online learning frameworks (Online Learning with Constraints, Regularized Online Optimization) and theoretical extensions to weaker function classes (Weakly DR-Submodular, Linearizable Optimization). The paper's treatment of biased gradients bridges first-order methods like Projection-Free Stochastic and derivative-free approaches, occupying a distinct niche between exact gradient computation and purely function-value-based optimization.

Among twelve candidates examined across three contributions, none were found to clearly refute the paper's claims. The Lyapunov framework extension examined two candidates with no refutable overlap; the 1/e approximation for convex sets with largest element examined ten candidates without finding prior work achieving this ratio for this constraint class; the quantum zero-order algorithms examined zero candidates. The limited search scope—twelve papers from semantic search and citation expansion—suggests these findings reflect the immediate neighborhood rather than exhaustive coverage. The novel constraint class (convex sets with largest element) appears particularly underexplored in the examined literature.

Based on the top-twelve semantic matches and taxonomy structure, the work appears to occupy a relatively uncrowded position within gradient estimation methods for DR-submodular optimization. The biased gradient setting and the specific constraint class represent departures from existing literature, though the limited search scope means potentially relevant work in adjacent optimization communities may not have been captured. The quantum acceleration component aligns with emerging trends but lacks direct comparators in the examined candidate set.

This paper presents **3 main contributions**, each analyzed against relevant prior work:

Contribution 1: Lyapunov framework extension for stochastic biased gradients

Description: The authors extend the existing Lyapunov framework, originally designed for exact gradients in DR-submodular maximization, to handle stochastic biased gradients. This extension characterizes how both bias and noise adversely affect algorithm performance.

This contribution was assessed against **2 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

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1. A single recipe for online submodular maximization with adversarial or stochastic constraints

URL: View paper

Brief Assessment

Online Submodular Recipe[3] focuses on online submodular maximization with cumulative convex constraints using Lyapunov drift analysis, not on extending Lyapunov frameworks for stochastic biased gradients in DR-submodular maximization as in the original paper.

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2. Zeroth-order stochastic approximation algorithms for DR-submodular optimization

URL: View paper

Brief Assessment

Zeroth-Order Stochastic[5] focuses on zeroth-order gradient estimation for DR-submodular optimization using coordinate-wise and randomized estimators, not on extending Lyapunov frameworks for biased gradients. The candidate's Lyapunov functions are used for convergence analysis of zeroth-order methods, not for characterizing bias and noise effects in stochastic biased gradient settings.

Contribution 2: 1/e approximation algorithm for convex sets with largest element

Description: The authors introduce a novel constraint class called convex sets with a largest element and propose an approximation algorithm achieving a 1/e ratio for non-monotone DR-submodular maximization. This result surpasses the known 1/4 hardness bound for general convex constraints by exploiting properties of the largest element.

This contribution was assessed against **10 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

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1. Nonmonotone Submodular Maximization Under Routing Constraints

URL: View paper

Brief Assessment

Routing Constraints[13] addresses nonmonotone submodular maximization with routing constraints (a k-system), not the continuous DR-submodular setting with convex sets containing a largest element that the original paper introduces.

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2. Non-monotone DR-submodular maximization over general convex sets

URL: View paper

Brief Assessment

General Convex Sets[17] focuses on general convex sets without the largest element property. The original paper introduces a novel constraint class (convex sets with largest element) that exploits specific structural properties to surpass the 1/4 hardness bound.

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3. Resolving the Approximability of Offline and Online Non-monotone DR-Submodular Maximization over General Convex Sets

URL: View paper

Brief Assessment

Approximability General Convex[18] focuses on general convex sets without a largest element, achieving 1/4(1-m) approximation. The original paper's novel constraint class (convex sets with a largest element) and its 1/e approximation are not addressed by this candidate.

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4. Continuous Non-monotone DR-submodular Maximization with Down-closed Convex Constraint

URL: View paper

Brief Assessment

Down-Closed Convex[14] focuses on down-closed convex constraints, not convex sets with a largest element. The candidate's 0.385 approximation for down-closed convex constraints is technically distinct from the original paper's 1/e approximation for the novel constraint class of convex sets with a largest element.

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5. Stochastic approximation algorithms for DR-submodular maximization with convex functional constraints

URL: View paper

Brief Assessment

Convex Functional Constraints[10] addresses DR-submodular maximization with convex functional constraints (expectation formulations), not the specific constraint class of 'convex sets with a largest element' introduced in the original paper. The candidate focuses on stochastic approximation algorithms for functional constraints rather than exploiting properties of a largest element in the constraint set.

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6. A unified approach for maximizing continuous DR-submodular functions

URL: View paper

Brief Assessment

Unified DR-Submodular[15] focuses on general convex sets, down-closed convex sets, and oracle access types, but does not address the specific constraint class of 'convex sets with a largest element' introduced in the original paper.

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7. Fast Approximation Algorithm for Non-Monotone DR-submodular Maximization under Size Constraint

URL: View paper

Brief Assessment

Non-Monotone Size Constraint[11] focuses on DR-submodular maximization under size constraints on integer lattices, not convex sets with a largest element. The constraint structures and problem formulations are fundamentally different.

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8. Linear Query Approximation Algorithms for Non-monotone Submodular Maximization under Knapsack Constraint

URL: View paper

Brief Assessment

Linear Query Knapsack[12] addresses non-monotone submodular maximization under knapsack constraints, not DR-submodular maximization under convex sets with a largest element. The constraint types and problem formulations are fundamentally different.

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9. Improved Parallel Algorithm for Non-Monotone Submodular Maximization under Knapsack Constraint

URL: View paper

Brief Assessment

Parallel Knapsack[19] addresses non-monotone submodular maximization under knapsack constraints with parallel algorithms, not the novel constraint class of convex sets with a largest element introduced in the original paper.

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10. Robust Approximation Algorithms for Non-Monotone k-Submodular Maximization Under a Knapsack Constraint

URL: View paper

Brief Assessment

k-Submodular Knapsack[16] addresses k-submodular maximization under knapsack constraints, which is a discrete optimization problem over k-sets. The original paper focuses on continuous DR-submodular maximization over convex sets with a largest element, a fundamentally different problem class with different mathematical structures and techniques.

Contribution 3: Quantum zero-order algorithms with improved complexity

Description: The authors develop quantum gradient estimation algorithms for zero-order DR-submodular maximization that achieve $O(\varepsilon-1)$ iteration complexity, matching classical first-order methods and demonstrating quantum acceleration over classical zero-order methods which require $O(\varepsilon-3)$ iterations.

This contribution was assessed against **0 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

Appendix: Text Similarity Detection

No high-similarity text segments were detected across any compared papers.

References

- [0] DR-Submodular Maximization with Stochastic Biased Gradients: Classical and Quantum Gradient Algorithms View paper
- [1] From linear to linearizable optimization: A novel framework with applications to stationary and non-stationary dr-submodular optimization View paper
- [2] Optimal continuous DR-submodular maximization and applications to provable mean field inference View paper
- [3] A single recipe for online submodular maximization with adversarial or stochastic constraints View paper
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- [5] Zeroth-order stochastic approximation algorithms for DR-submodular optimization View paper
- [6] Stochastic Block-Coordinate Gradient Projection Algorithms for Submodular Maximization View paper
- [7] Regularized online DR-submodular optimization View paper
- [8] Information Content of Big Data View paper
- [9] A Unified Approach for Maximizing Continuous Î³-weakly DR-submodular Functions View paper
- [10] Stochastic approximation algorithms for DR-submodular maximization with convex functional constraints View paper
- [11] Fast Approximation Algorithm for Non-Monotone DR-submodular Maximization under Size Constraint View paper
- [12] Linear Query Approximation Algorithms for Non-monotone Submodular Maximization under Knapsack Constraint View paper
- [13] Nonmonotone Submodular Maximization Under Routing Constraints View paper
- [14] Continuous Non-monotone DR-submodular Maximization with Down-closed Convex Constraint View paper
- [15] A unified approach for maximizing continuous DR-submodular functions View paper
- [16] Robust Approximation Algorithms for Non-Monotone k-Submodular Maximization Under a Knapsack Constraint View paper
- [17] Non-monotone DR-submodular maximization over general convex sets View paper
- [18] Resolving the Approximability of Offline and Online Non-monotone DR-Submodular Maximization over General Convex Sets View paper
- [19] Improved Parallel Algorithm for Non-Monotone Submodular Maximization under Knapsack Constraint View paper