Novelty Assessment Report

Paper: $\boldsymbol{\partial^\infty}$-Grid: Differentiable Grid Representations for Fast and Accurate Solutions to Differential Equations
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Abstract

We present a novel differentiable grid-based representation for efficiently solving differential equations (DEs). Widely used architectures for neural solvers, such as sinusoidal neural networks, are coordinate-based MLPs that are, both, computationally intensive and slow to train. Although grid-based alternatives for implicit representations (e.g., Instant-NGP and K-Planes) train faster by exploiting signal structure, their reliance on linear interpolation restricts their ability to compute higher-order derivatives, rendering them unsuitable for solving DEs. In contrast, our approach overcomes these limitations by combining the efficiency of feature grids with radial basis function interpolation, which is infinitely often differentiable. To effectively capture high-frequency solutions and enable stable and faster computation of global gradients, we introduce a multi-resolution decomposition with co-located grids. Our proposed representation, $\boldsymbol{\partial^\infty}$-Grid, is trained implicitly using the differential equations as loss functions, enabling accurate modeling of physical fields. We validate $\boldsymbol{\partial^\infty}$-Grid on a variety of tasks, including Poisson equation for image reconstruction, the Helmholtz equation for wave fields, and the Kirchhoff-Love boundary value problem for cloth simulation. Our results demonstrate a 5–20× speed-up over coordinate-based MLP-based methods, solving differential equations in seconds or minutes while maintaining comparable accuracy and compactness.

Core Task Landscape

This paper addresses: **Solving Differential Equations with Neural Representations**

A total of **50 papers** were analyzed and organized into a taxonomy with **18 categories**.

Taxonomy Overview

The research landscape has been organized into the following main categories:

- **Neural Ordinary Differential Equations (Neural ODEs)**
- **Physics-Informed Neural Networks (PINNs) for PDEs**
- **Neural Operator Learning for PDEs**
- **Neural Network Representations and Approximation Theory**
- **Specialized Topics and Applications**

Complete Taxonomy Tree

- Solving Differential Equations with Neural Representations Survey Taxonomy
- Neural Ordinary Differential Equations (Neural ODEs)
  - Core Neural ODE Architectures and Extensions (4 papers)
  - [3] On neural differential equations (Kidger, 2022) View paper
  - [7] Neural ordinary differential equations (Chen, 2018) View paper
  - [8] nmODE: neural memory ordinary differential equation (Zhang Yi, 2023) View paper
  - [25] Anamnesic Neural Differential Equations with Orthogonal Polynomial Projections (De Brouwer, 2023) View paper
  - Specialized Neural ODE Variants (5 papers)
  - [1] Stiff neural ordinary differential equations (Suyong Kim, 2021) View paper
  - [2] Learning neural event functions for ordinary differential equations (Chen, 2020) View paper
  - [10] Neural integro-differential equations (Zappala, 2023) View paper
  - [26] Neural network method: delay and system of delay differential equations (Shagun Panghal, 2022) View paper
  - [43] Neural adaptive delay differential equations (Chao Zhou, 2025) View paper
  - Neural ODE Training and Optimization (2 papers)
  - [9] Learning differential equations that are easy to solve (Jacob Kelly, 2020) View paper
  - [23] Distribution learning via neural differential equations: minimal energy regularization and approximation theory (Marzouk, 2025) View paper
  - Neural ODE Applications and System Identification (5 papers)
  - [18] Comprehensive review of neural differential equations for time series analysis (Oh, 2025) View paper
  - [21] Neural ordinary differential equations for nonlinear system identification (Aowabin Rahman, 2022) View paper
  - [41] Neural ordinary differential equation based sequential image registration for dynamic characterization (Wu, 2024) View paper
  - [42] Inferring latent dynamics underlying neural population activity via neural differential equations (Timothy Kim, 2021) View paper
  - [48] Stable Neural Stochastic Differential Equations in Analyzing Irregular Time Series Data (Oh, 2024) View paper
- Physics-Informed Neural Networks (PINNs) for PDEs
  - Core PINN Methodologies and Formulations (5 papers)
  - [4] Using NeuralPDE.jl to solve differential equations (Daria M. Belicheva, 2025) View paper

  - [11] A unified deep artificial neural network approach to partial differential equations in complex geometries (Jens Berg, 2018) View paper
  - [22] An introduction to neural network methods for differential equations (N. Yadav, 2015) View paper
  - [28] Three ways to solve partial differential equations with neural networksâ□□A review (Blechschmidt, 2021) View paper
  - [46] Solving Differential Equations with Physics-Informed Neural Networks (Chenghao Dong, 2025) View paper
  - PINN Enhancements and Adaptive Strategies (3 papers)
  - [19] Physics-informed neural networks with residual/gradient-based adaptive sampling methods for solving partial differential equations with sharp solutions (Zhiping Mao, 2023) View paper
  - [39] Partitioned neural network approximation for partial differential equations enhanced with Lagrange multipliers and localized loss functions (Deok-Kyu Jang, 2023) View paper
  - [40] Solving Differential Equations with Constrained Learning (Chamon, 2024) View paper
  - Alternative Network Architectures for PDE Solving (3 papers)
  - [24] Solving Poisson Equations using Neural Walk-on-Spheres (Berner, 2024) View paper
  - [45] Solving partial differential equations with sampled neural networks (Datar, 2024) View paper
  - [49] Physics-informed radial basis network (PIRBN): A local approximating neural network for solving nonlinear partial differential equations (Jinshuai Bai, 2023) View paper
- Neural Operator Learning for PDEs
  - Operator Network Architectures (2 papers)
  - [20] Neural Operator: Graph Kernel Network for Partial Differential Equations (Li, 2020) View paper
  - [37] Fourier Neural Operator for Parametric Partial Differential Equations (Li, 2020) View paper
  - Specialized Operator Learning Approaches (3 papers)
  - [14] In-context operator learning with data prompts for differential equation problems (Liu Yang, 2023) View paper
  - [38] Vectorized conditional neural fields: A framework for solving time-dependent parametric partial differential equations (Kalimuthu Marimuthu, 2024) View paper
  - [47] Transferable Neural Networks for Partial Differential Equations (Zezhong Zhang, 2023) View paper
- Neural Network Representations and Approximation Theory
  - General Neural Approximation Methods for DEs (7 papers)
  - [5] Neural network representation for ordinary differential equations (Anna Golovkina, 2022) View paper
  - [6] A neural network approach for solving nonlinear differential equations of Laneâ□□Emden type (K. Parand, 2023) View paper
  - [12] Neural network for solving differential equations (Khalil M, 2025) View paper
  - [32] Deep neural network for system of ordinary differential equations: Vectorized algorithm and simulation (Tamirat Temesgen Dufera, 2021) View paper
  - [35] Solving differential equations with unsupervised neural networks (D. Parisi, 2003) View paper
  - [44] Artificial neural network approach for solving fuzzy differential equations (Sohrab Effati, 2010) View paper
  - [50] Neuralâ□□networkâ□□based approximations for solving partial differential equations (M. Dissanayake, 1994) View paper
  - Advanced Representation Methods ★ (2 papers)
  - [0] $\boldsymbol{\partial^\infty}$-Grid: Differentiable Grid Representations for Fast and Accurate Solutions to Differential Equations (Anon et al., 2026) View paper
  - [16] Signal processing for implicit neural representations (Xu, 2022) View paper
- Specialized Topics and Applications
  - Inverse Problems and Parameter Identification (1 papers)
  - [29] The neural network approach to inverse problems in differential equations (Xu Kai-Lai, 2019) View paper
  - Uncertainty Quantification and Robustness (1 papers)
  - [15] NeuralUQ: A comprehensive library for uncertainty quantification in neural differential equations and operators (Zongren Zou, 2024) View paper
  - Coupled and Fractional Differential Equations (2 papers)
  - [13] Advanced neural network approaches for coupled equations with fractional derivatives (Suleman Alfalqi, 2024) View paper
  - [36] NeuPDE: Neural network based ordinary and partial differential equations for modeling time-dependent data (Sun, 2020) View paper
  - Software Libraries and Computational Tools (2 papers)
  - [30] Diffeqflux. jl-A julia library for neural differential equations (Rackauckas, 2019) View paper
  - [34] Neurodiffeq: A python package for solving differential equations with neural networks (Chen Fei-yu, 2020) View paper
  - Comparative Studies and Benchmarking (2 papers)
  - [27] Review of neural network-based methods for solving partial differential equations (Z Wenshu, 2022) View paper
  - [31] A comprehensive and FAIR comparison between MLP and KAN representations for differential equations and operator networks (Shukla, 2024) View paper
  - Neural Programming and Symbolic Approaches (1 papers)
  - [33] Towards solving differential equations through neural programming (F Arabshahi, 2018) View paper
  - Graph Neural Networks for Differential Equations (1 papers)
  - [17] Graph odes and beyond: A comprehensive survey on integrating differential equations with graph neural networks (Zewen Liu, 2025) View paper

Narrative

Core task: Solving differential equations with neural representations. The field has evolved into several major branches that reflect different modeling philosophies and application domains. Neural Ordinary Differential Equations (Neural ODEs[7]) treat the hidden state dynamics of deep networks as continuous-time flows, enabling memory-efficient training and adaptive computation. Physics-Informed Neural Networks (PINNs[46]) embed known physical laws directly into the loss function, allowing networks to approximate solutions to partial differential equations without large labeled datasets. Neural Operator Learning (e.g., Fourier Neural Operator[37]) shifts focus from point-wise approximation to learning mappings between entire function spaces, offering a data-driven route to surrogate modeling for complex PDEs. Meanwhile, Neural Network Representations and Approximation Theory investigates the expressive power and convergence guarantees of these architectures, and Specialized Topics cover extensions such as stochastic, delay, and inverse problems. Together, these branches span the spectrum from theoretical foundations to practical solvers for scientific computing.

Recent work has explored trade-offs between expressiveness, computational efficiency, and adherence to physical constraints. Many studies in the Neural ODE line (Stiff Neural ODEs[1], Neural Event Functions[2]) address numerical stability and event-driven dynamics, while operator learning methods (Neural Operator[20], In-context Operator Learning[14]) emphasize generalization across parameter regimes. Within the Advanced Representation Methods cluster, Differentiable Grid[0] introduces a structured spatial discretization that bridges classical finite-difference schemes and implicit neural representations, offering a middle ground between mesh-based and mesh-free approaches. This contrasts with purely coordinate-based methods like Signal Processing INR[16], which encode signals as continuous functions without explicit grid structure. By combining differentiable grids with neural parameterizations, Differentiable Grid[0] aims to retain interpretability and computational tractability while leveraging the flexibility of learned representations, positioning itself at the intersection of classical numerical analysis and modern deep learning for PDEs.

Related Works in Same Category

The following **1 sibling papers** share the same taxonomy leaf node with the original paper:

1. Signal processing for implicit neural representations

Authors: Xu, Dejia, Wang, Peihao, Dejia Xu, et al. (12 authors total) | **Year/Venue**: 2022 | **URL**: View paper

Abstract

Implicit Neural Representations (INRs) encoding continuous multi-media data via multi-layer perceptrons has shown undebatable promise in various computer vision tasks. Despite many successful applications, editing and processing an INR remains intractable as signals are represented by latent parameters of a neural network. Existing works manipulate such continuous representations via processing on their discretized instance, which breaks down the compactness and continuous nature of INR. In this...

Relationship Analysis

Both papers belong to the Advanced Representation Methods category, focusing on specialized neural representations for solving differential equations and signal processing. They share the use of grid-based and implicit neural representations with emphasis on differentiability; the original paper (∂∞-Grid) specifically addresses solving DEs using radial basis function interpolation on multi-resolution grids, while the candidate paper (INSP) focuses on signal processing operations on existing INRs through high-order differential operators. The key difference is that ∂∞-Grid solves DEs directly to model physical fields, whereas INSP processes already-fitted INRs for image/signal manipulation tasks without explicitly solving DEs.

Contributions Analysis

Overall novelty summary. The paper proposes ∂∞-Grid, a differentiable grid-based representation that combines feature grids with radial basis function (RBF) interpolation for solving differential equations. This work resides in the 'Advanced Representation Methods' leaf of the taxonomy, which contains only two papers total. This leaf sits within the broader 'Neural Network Representations and Approximation Theory' branch, indicating a focus on representation design rather than specific solver architectures. The sparse population of this leaf suggests the paper addresses a relatively underexplored niche: bridging efficient grid-based implicit representations with the smoothness requirements of DE solving.

The taxonomy reveals that most neural DE solving work concentrates in three major branches: Neural ODEs (17 papers across four leaves), PINNs (11 papers across three leaves), and Neural Operator Learning (5 papers across two leaves). The 'Advanced Representation Methods' leaf neighbors 'General Neural Approximation Methods for DEs' (7 papers), which covers foundational feedforward and trial solution approaches. The sibling paper in this leaf (Differentiable Grid) also explores structured spatial discretization. The paper's focus on multi-resolution grids and RBF interpolation distinguishes it from coordinate-based MLPs prevalent in PINNs and from operator learning methods that map between function spaces.

Among 29 candidates examined, the analysis identified varying novelty across contributions. The core ∂∞-Grid representation (10 candidates examined, 0 refutable) and multi-resolution decomposition (9 candidates, 0 refutable) appear to have limited direct prior work within this search scope. However, the implicit training framework using DEs as loss functions (10 candidates examined, 4 refutable) shows substantial overlap with existing PINN methodologies. This suggests the representation architecture itself may be more novel than the training paradigm, which builds on established physics-informed learning principles widely adopted in the field.

Based on this limited search of 29 semantically similar papers, the work appears to occupy a relatively sparse research direction within neural DE solving. The representation design shows fewer overlaps than the training methodology, though the small candidate pool and focused taxonomy leaf prevent definitive claims about absolute novelty. The analysis captures top-K semantic matches and does not constitute an exhaustive literature review across all grid-based or RBF-based neural methods.

This paper presents **3 main contributions**, each analyzed against relevant prior work:

Contribution 1: ∂∞-Grid: differentiable grid-based representation combining feature grids with RBF interpolation

Description: The authors introduce ∂∞-Grid, a new representation that combines the computational efficiency of feature grids with radial basis function (RBF) interpolation to enable infinite differentiability, overcoming limitations of existing grid-based methods that rely on linear interpolation and cannot compute higher-order derivatives needed for solving differential equations.

This contribution was assessed against **10 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

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1. On a high-order Gaussian radial basis function generated Hermite finite difference method and its application

URL: View paper

Brief Assessment

Gaussian RBF Hermite[65] focuses on high-order Hermite finite difference methods for numerical PDE solving, not on differentiable neural representations combining feature grids with RBF interpolation for machine learning applications. The candidate paper addresses a fundamentally different problem domain (numerical analysis) than the original paper's neural differential equation solver.

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2. An effective high-order five-point stencil, based on integrated-RBF approximations, for the first biharmonic equation and its applications in fluid dynamics

URL: View paper

Brief Assessment

Five-Point Stencil RBF[68] focuses on numerical discretization schemes for solving the first biharmonic equation using integrated RBF approximations on five-point stencils, not on differentiable feature grid representations for neural PDE solvers with automatic differentiation.

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3. Fast radial basis functions for engineering applications

URL: View paper

Brief Assessment

Fast Radial Basis[61] focuses on mesh deformation applications in engineering contexts. The candidate's extremely limited text does not provide sufficient detail about grid-based representations, feature learning, or neural differential equation solving to challenge the novelty of ∂∞-Grid's specific combination of multi-resolution feature grids with RBF interpolation for solving PDEs.

4. A Gaussian type radial basis function method to solve Black-Scholes equation

URL: View paper

Brief Assessment

Gaussian RBF Black-Scholes[66] applies RBF methods to solve the Black-Scholes equation for financial options pricing, not to create differentiable grid-based representations for general differential equations or neural field learning as in the original paper.

5. Accuracy of radial basis function interpolation and derivative approximations on 1-D infinite grids

URL: View paper

Brief Assessment

RBF Interpolation Accuracy[70] focuses on analyzing RBF interpolation accuracy on 1-D infinite grids for derivative approximations, not on developing a differentiable grid-based representation for solving differential equations. The candidate paper is a mathematical analysis of existing RBF methods, while the original proposes a novel neural solver architecture.

6. Hermite type radial basis function-based differential quadrature approach allows for free vibration beams for higher order equations

URL: View paper

Brief Assessment

Hermite RBF Vibration[62] focuses on radial basis functions for solving beam vibration equations in structural mechanics, not on differentiable grid-based representations for neural PDE solvers or feature grids combined with RBF interpolation for computing higher-order derivatives in machine learning contexts.

7. Radial basis function-differential quadrature-based physics-informed neural network for steady incompressible flows

URL: View paper

Brief Assessment

RBF Physics-Informed[69] uses RBF differential quadrature for derivative calculation in physics-informed neural networks for fluid dynamics, not for creating a differentiable grid-based representation with infinite differentiability for general differential equation solving.

8. Generalized moving least squares vs. radial basis function finite difference methods for approximating surface derivatives

URL: View paper

Brief Assessment

Moving Least Squares[63] focuses on surface derivative approximation methods for computational geometry, not on differentiable grid-based representations for solving differential equations. The candidate paper's full text context is not available (marked as 'n/a'), preventing detailed comparison of technical approaches.

9. Adaptive radial basis function methods for time dependent partial differential equations

URL: View paper

Brief Assessment

Adaptive RBF Methods[67] focuses on adaptive collocation point selection for time-dependent PDEs using traditional RBF interpolation methods, not on combining feature grids with RBF interpolation for neural differential equation solvers or enabling infinite differentiability in grid-based representations.

10. Radial basis function methods for the Rosenau equation and other higher order PDEs

URL: View paper

Brief Assessment

Rosenau Equation RBF[64] focuses on solving the Rosenau equation and other higher-order PDEs using RBF collocation methods with fictitious point and resampling techniques. The original paper addresses neural differential equation solvers with differentiable feature grids for implicit field representation, which is a fundamentally different application domain and architectural approach.

Contribution 2: Multi-resolution decomposition with co-located grids

Description: The authors propose a multi-resolution decomposition approach using co-located grids to effectively capture high-frequency solutions and enable stable and faster computation of global gradients in their grid-based representation.

This contribution was assessed against **9 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

1. Offshore Wind Energy Prediction Using Machine Learning with Multi-Resolution Inputs

URL: View paper

Brief Assessment

Wind Energy Prediction[72] focuses on wind energy forecasting using multi-resolution numerical weather prediction (NWP) models as inputs to machine learning systems, not on grid-based neural representations for solving differential equations with co-located feature grids.

2. Analysis of Registration Requirements and Techniques for Imaging Sensor Suites on Uninhabited Vehicles

URL: View paper

Brief Assessment

Registration Imaging Sensors[77] focuses on sensor registration and alignment techniques for imaging systems on uninhabited vehicles. The sparse mentions of 'multiresolution' and 'co-located' in the candidate appear in different contexts (sensor co-location, control grids) rather than addressing the original paper's neural differential equation solver with multi-resolution feature grids for capturing high-frequency solutions and enabling stable gradient computation.

3. M2NO: An Efficient Multi-Resolution Operator Framework for Dynamic Multi-Scale PDE Solvers

URL: View paper

Brief Assessment

M2NO[71] uses a multiwavelet-based multigrid structure for multi-resolution analysis in neural operators for PDEs, which is architecturally distinct from the original paper's co-located feature grids with RBF interpolation for implicit neural representations.

4. Multiresolution hierarchies on unstructured triangle meshes

URL: View paper

Brief Assessment

Multiresolution Triangle Meshes[74] focuses on decomposing triangle meshes into hierarchical representations for geometry processing, not on grid-based neural representations for solving differential equations with co-located feature grids and RBF interpolation.

5. An efficient multi-scale waveform inversion method in Laplace-Fourier domain

URL: View paper

Brief Assessment

Multi-Scale Waveform Inversion[73] focuses on seismic waveform inversion in the Laplace-Fourier domain, not neural differential equation solvers with feature grids for capturing high-frequency solutions through RBF interpolation and co-located multi-resolution grids.

6. Flexible voxels for motion-aware videography

URL: View paper

Brief Assessment

Flexible Voxels[75] addresses multi-resolution sampling in the spatio-temporal domain for videography, not spatial feature grids for neural differential equation solving. The technical contexts are fundamentally different.

7. Multi-Grid Schemes for Multi-Scale Coordination of Energy Systems

URL: View paper

Brief Assessment

Multi-Grid Energy Systems[76] focuses on multi-scale coordination of energy systems using multi-grid computing schemes for spatial and temporal decomposition in optimization problems. The original paper addresses neural differential equation solvers with feature grids for capturing high-frequency solutions in physical fields, which is a fundamentally different domain and technical approach.

8. A multi-scale decomposition and component-wise differentiated fusion strategy for water storage change prediction: a case study of the North China Plain

URL: View paper

Brief Assessment

Water Storage Prediction[78] focuses on water storage change prediction using decomposition strategies for hydrological forecasting, not neural differential equation solvers with grid-based representations for capturing high-frequency solutions in physical simulations.

9. Practical Facial Geometry and Appearance Capture at Home

URL: View paper

Brief Assessment

Facial Geometry Capture[79] applies multi-resolution hash grids for facial geometry representation in computer vision, while the original paper proposes multi-resolution co-located grids specifically for solving differential equations with RBF interpolation. The technical contexts and objectives differ fundamentally.

Contribution 3: Implicit training framework using differential equations as loss functions

Description: The authors develop a training approach where the differential equations themselves serve as loss functions for implicit optimization, enabling accurate modeling of physical fields governed by these equations.

This contribution was assessed against **10 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

1. Physics-Informed Quantum Machine Learning: Solving nonlinear differential equations in latent spaces without costly grid evaluations

URL: View paper

Brief Assessment

Quantum Machine Learning[60] focuses on quantum algorithms for solving differential equations in quantum latent spaces using state overlaps, not grid-based neural representations. The original paper develops a differentiable feature grid with RBF interpolation for classical neural solvers, which is fundamentally different from quantum circuit-based approaches.

2. A finite element-based physics-informed operator learning framework for spatiotemporal partial differential equations on arbitrary domains

URL: View paper

Prior Art Analysis

Finite Element Operator[54] demonstrates that implicit training using differential equations as loss functions was already established in prior work. The candidate paper explicitly describes training networks using physics-informed loss functions derived from governing PDEs, where 'the training follows a physics-informed loss function constructed based on the finite element discretization of the heat equation' and networks are 'trained based on physics-based loss functions from bvps'. This approach predates the original paper's contribution, as the candidate was published in 2025 while discussing established methods from earlier years.

Evidence

Evidence 1 - **Rationale**: Both papers describe training networks using differential equations as loss functions. The candidate explicitly states this approach was already established in prior work [69], demonstrating this was not a novel contribution by the original paper. - **Original**: our proposed representation,∂ ∞-grid, is trained implicitly using the differential equations as loss functions, enabling accurate modelling of physical fields - **Candidate**: the training follows a physics-informed loss function constructed based on the finite element discretization of the heat equation [69], thereby making it unsupervised learning without labeled data

Evidence 2 - **Rationale**: The candidate paper describes the established concept of training neural networks using PDEs as loss functions, attributing this to prior work (PINNs [21]), which refutes the novelty claim of the original paper's implicit training framework. - **Original**: the optimisation of network weights can emulate the process of solving physical equations; we refer to such networks as neural solvers for brevity - **Candidate**: nns trained based on physics-based loss functions from bvps are called physics-informed neural networks (pinns) [21]. the key idea is to incorporate governing pdes directly into the loss functions of nns with the power of automatic differentiation

Evidence 3 - **Rationale**: Both papers propose frameworks for solving differential equations using physics-informed loss functions. The candidate's approach of incorporating physics into the loss function through discretized weak formulation demonstrates that implicit training using differential equations as loss was already an established approach. - **Original**: we propose a novel differentiable grid-based representation for efficiently solving differential equations (des) - **Candidate**: we propose a novel finite element-based physics-informed operator learning framework that allows for predicting spatiotemporal dynamics governed by partial differential equations (pdes). the galerkin discretized weak formulation is employed to incorporate physics into the loss function

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3. Unsupervised learning with physics informed graph networks for partial differential equations

URL: View paper

Brief Assessment

Physics Informed Graphs[56] focuses on graph-based methods for solving PDEs like Poisson's equation, while the original paper presents a grid-based representation with RBF interpolation. The candidate's limited context does not demonstrate prior work on the specific implicit training framework combining feature grids with differential equation losses.

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4. PiRD: Physics-informed Residual Diffusion for Flow Field Reconstruction

URL: View paper

Brief Assessment

PiRD[53] focuses on diffusion models for fluid dynamics data enhancement and uses physics-based insights in the objective function, but does not propose an implicit training framework where differential equations themselves serve as loss functions for optimization. The candidate addresses data fidelity enhancement in fluid dynamics, while the original paper presents a general neural solver framework for various differential equations.

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5. Universal differential equations for scientific machine learning

URL: View paper

Prior Art Analysis

Universal Differential Equations[51] demonstrates that using differential equations as loss functions for implicit optimization was established prior to the original paper. The candidate explicitly describes training universal differential equations by formulating the differential equation residual as the loss function, where the solution field is optimized by minimizing this residual over the domain. This approach directly incorporates physical constraints into the training process through the differential equation formulation itself, which serves as the optimization objective.

Evidence

Evidence 1 - **Rationale**: Both papers describe training by minimizing a cost function defined by the differential equation itself, demonstrating that this implicit training approach was established in the candidate paper. - **Original**: our proposed representation,∂ ∞-grid, is trained implicitly using the differential equations as loss functions, enabling accurate modelling of physical fields. - **Candidate**: training a ude amounts to minimizing a cost function c(θ) defined on uθ(t), the current solution to the differential equation with respect to the choice of parameters θ.

Evidence 2 - **Rationale**: Both papers describe computing derivatives of the solution with respect to parameters for training, but the candidate demonstrates this was an established approach in the UDE framework before the original paper. - **Original**: we propose to compute gradients, jacobians, and laplacians of the decoded interpolated features with respect to query samplesxusing automatic differentiation (autograd). these derivatives are essential for solving differential equations. - **Candidate**: this requires the calculation of dc dθ which by the chain rule amounts to calculating du dθ. thus the problem of efficiently training a ude reduces to calculating gradients of the differential equation solution with respect to parameters.

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6. A practical approach to flow field reconstruction with sparse or incomplete data through physics informed neural network

URL: View paper

Brief Assessment

Flow Field Reconstruction[59] focuses on reconstructing flow fields with sparse data using physics-informed neural networks, not on developing a general implicit training framework with differential equations as loss functions for diverse physical field modeling.

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7. Measurement-Physic Constrained Neural Network for Multi-Field Reconstruction of PMC

URL: View paper

Brief Assessment

Multi-Field Reconstruction PMC[57] focuses on electromagnetic field reconstruction in permanent magnet couplers using measurement-constrained networks, not a general implicit training framework for differential equations as loss functions.

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8. Variational problems and partial differential equations on implicit surfaces

URL: View paper

Brief Assessment

Variational Implicit Surfaces[58] focuses on solving variational problems and PDEs on implicit surfaces using classical numerical methods and level set representations, not on neural implicit training frameworks where differential equations serve as loss functions for learning physical fields.

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9. Implicit Neural Differential Model for Spatiotemporal Dynamics

URL: View paper

Prior Art Analysis

Implicit Neural Differential[52] demonstrates that implicit training frameworks using differential equations as loss functions were already established in the scientific machine learning literature. The candidate paper explicitly describes training implicit neural networks where 'the training of pindiff model over dataset d = {˜xi, ϕi}n i=1 is formulated as a pde-constrained optimization problem' where differential equations serve as constraints in the optimization. This formulation directly uses differential equations as part of the loss/optimization framework, similar to the original paper's approach of using 'the differential equations themselves serve as loss functions for implicit optimization.'

Evidence

Evidence 1 - **Rationale**: Both papers address the challenge of creating differentiable frameworks for solving differential equations. The candidate's implicit neural differential model predates the original submission and demonstrates that implicit training with differential equations as constraints was already established. - **Original**: we propose a novel differentiable grid-based representation for efficiently solving differential equations (des). ... our approach overcomes these limitations by combining the efficiency of feature grids with radial basis function interpolation, which is infinitely differentiable. - **Candidate**: by employing implicit neural network layers, our framework mitigates error accumulation and significantly enhances numerical stability and accuracy, enabling reliable long-term simulations. however, adopting implicit neural architectures within differentiable frameworks introduces considerable compu...

10. Learning to control pdes with differentiable physics

URL: View paper

Prior Art Analysis

Differentiable Physics Control[55] demonstrates prior work that uses differential equations directly as loss functions for training neural networks. The paper explicitly describes training networks where 'the pde residual over the spatio-temporal domain' serves as the loss function, and states that 'differentiable physics losses solve these problems by allowing the agent to be directly optimized for the desired objective.' This approach of using PDEs as loss functions for implicit optimization predates the original paper's claimed novelty, as the candidate paper was published at ICLR 2020.

Evidence

Evidence 1 - **Rationale**: The candidate paper describes 'differentiable physics losses' that directly optimize for objectives defined by differential equations, which is conceptually identical to the original paper's claim of using differential equations as loss functions for implicit training. - **Original**: we propose a feature grid-based representation for physical fields and a formulation recovering fields from differential equations (sec. 3.1). - **Candidate**: differentiable physics losses solve these problems by allowing the agent to be directly optimized for the desired objective (eq. 4). unlike supervised losses, differentiable physics losses require a differentiable solver to backpropagate the gradients through the simulation.

Evidence 2 - **Rationale**: Both papers train neural networks by optimizing loss functions derived from differential equations. The candidate's approach of using PDE residuals as training objectives demonstrates that this implicit training framework existed prior to the original paper's submission. - **Original**: recalling our goal of optimising foruin problems of the form eq. (1), we define the loss function as: l(f,θ) = z ω f(.)dx= x i∈d f xi,u(x i),∇xiu(xi),∇2 xiu(xi), . . .;g(xi) - **Candidate**: we present a novel deep learning approach that can learn to represent solution manifolds for a given physical environment, and is orders of magnitude faster than iterative optimization techniques. the core of our method is a hierarchical predictor-corrector scheme that temporally divides the problem...

Appendix: Text Similarity Detection

No high-similarity text segments were detected across any compared papers.

References

- [0] $\boldsymbol{\partial^\infty}$-Grid: Differentiable Grid Representations for Fast and Accurate Solutions to Differential Equations View paper
- [1] Stiff neural ordinary differential equations View paper
- [2] Learning neural event functions for ordinary differential equations View paper
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